## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

M.Sc. DEGREE EXAMINATION - PHYSICS

SECOND SEMESTER - APRIL 2015
PH 2816-QUANTUM MECHANICS - I
Dept. No. $\square$ Max. : 100 Marks
Date : 21/04/2015
Time : 01:00-04:00

PART - A
Answer all the questions.
( $10 \times 2$ = 20 Marks)

1. Define the terms (i) inner product and (ii) orthonormalisation
2. Show that $\mathrm{i} \frac{d}{d x}$ is aHermitian.
3. Show that the fundamental commutation relation $\left[\mathrm{x}, \mathrm{p}_{\mathrm{x}}\right]=\mathrm{i} \hbar$ remains unchanged under unitary transformation.
4. If A', B', C' are related to $A, B, C$ respectively through unitary transformation, show that the commutation relation $[\mathrm{A}, \mathrm{B}]=\mathrm{iC}$ is invariant under unitary transformation.
5. Show that $<0\left|\left(a^{\dagger} a^{\dagger} a^{\dagger}\right)\right| 0>=0$, where $a$ and $a^{\dagger}$ are the lowering and raising operators respectively.
6. Show that the ground state of the hydrogen atom will not exhibit a first order Stark effect.
7. Establish any two properties of Pauli matrices.
8. If $\mathrm{j}_{1}=1$ and $\mathrm{j}_{2}=1 / 2$, what are the allowed values of resultant $\mathbf{J}=\mathbf{J}_{\mathbf{1}}+\mathbf{J}_{\mathbf{2}}$
9. What is the principle of partial wave analysis?
10. Define differential cross section and total cross section for scattering.

## PART -B

Answer any four questions.
( $4 \times 7.5$ = 30 Marks)
11. Show that (i) operators having common set of eigenfunctions commute.
(ii) Commuting operators have a common set of eigenfunctions.
12. Usingortho normal eigenkets $\mid \mathrm{s}, \mathrm{m}_{\mathrm{s}}>$ with $\mathrm{s}=1 / 2$, obtain $\mathrm{S}_{\mathrm{x}}, \mathrm{S}_{\mathrm{y}}$ and $\mathrm{S}_{\mathrm{z}}$ matrices.
13. Obtain the first order correction to energy of an anharmonic oscillator for a perturbation of the form $\mathrm{bx}^{4}$.
14. Assuming that $\left\langle\mathrm{j}_{1} \mathrm{j}_{2} \mid \mathrm{j}_{1}+\mathrm{j}_{2}, \mathrm{j}_{1}+\mathrm{j}_{2}\right\rangle=+1$, show that $\left\langle\mathrm{j}_{1}, \mathrm{j}_{2}-1 \mid \mathrm{j}_{1}+\mathrm{j}_{2}-1, \mathrm{j}_{1}+\mathrm{j}_{2}-1\right\rangle=\sqrt{ }\left(\frac{j_{1}}{j_{1}+j_{2}}\right)$ and $<\mathrm{j}_{1}-1, \mathrm{j}_{2} \mid \mathrm{j}_{1}+\mathrm{j}_{2}-1, \mathrm{j}_{1}+\mathrm{j}_{2}-1>=-\sqrt{ }\left(\frac{j_{2}}{j_{1}+j_{2}}\right)$
15. Establish the commutation relationsof $J_{+}$with $J^{2}, J_{z}, J_{x}, J_{y}$.
16. Relate the differential scattering cross-section in the laboratory coordinate system with that in the center of mass coordinate system.

## PART - C

Answer any four questions.
17. Solve graphically the eigenvalue problem of a particle in a square-well potential with finite walls.
18. With necessary theory, explain quantum mechanical tunnelling.
19. (a) Starting from momentum representation, obtain the form of operator for position coordinate.
(b) Starting form coordinate representation, obtain the form of operator for momentum representation.
20. Explain the formation of the hydrogen molecule using variational method.
21. Obtain the matrix forms for $\mathrm{J}^{2}, \mathrm{~J}_{z}, \mathrm{~J}_{\mathrm{x}}, \mathrm{J}_{\mathrm{y}}, \mathrm{J}_{+}$and $\mathrm{J}_{-}$,when $\mathrm{j}=3 / 2$.
22. Discuss scattering by a central potential using partial wave analysis. Derive an expression for scattering cross-section and hence prove the optical theorem.

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